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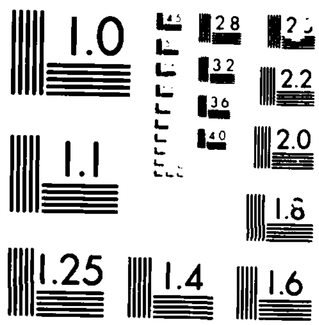

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ON THE ISOMORPHISM BETWEEN GAUGE GROUPS BEFORE AND AFTER RENORMALIZATION IN THE PRESENCE OF ABEL SUBGROUPS AND HIGGS FIELDS

by

Rong Wang

FOREIGN TECHNOLOGY DIVISION

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On The Isomorphism Between Gauge Groups Before And After Renormalization In The Presence Of Abel Subgroups And Higgs Fields

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ABSTRACT

We give a rigorous proof on the isomorphism between gauge groups before and after renormalization, in the presence of Abel subgroups and Higgs fields.

I.

The gauge groups which contain Abel subgroups (The W-S model is a special case of these groups) are more complicated than the gauge groups which do not contain Abel subgroups. This complicated case was never discussed by 't Hooft and Veltman [1], and by Lee and Zinn-Justin [2] in their early renormalization studies. Later, although Ross and Taylor [3] studied the renormalization of the W-S model and the gauge invariance after the renormalization, their discussion was not rigorous. In fact, their results were accurate only within two orders of perturbation. By using the identical equations of Slavnov, Julve and Tonin [4] found out that gauge groups before and after renormalization were isomorphic even in the presence of Abel subgroups. Their method is very similar to Zinn-Justin's discussion where the Abel subgroups were not presented.[5] But this

method, as pointed out by Zinn-Justin [5] and Taylor [6], requires a strict definition of S_n^0. Namely, for each order (i.e. each n, n represents n loops), $\tilde{S}_n^0 * \tilde{S}_n^0 = 0$ must be satisfied. To achieve this requirement, $\tilde{S}_n^0$ after n loops of renormalization and $\tilde{S}_{n+1}^0$ after n+1 loops of renormalization should satisfy the following relation:

$$\tilde{S}_{n+1}^0 = \tilde{S}_n^0 - \Gamma_{\underset{(n+1)}{\text{div}}}(S_n^0) + o(\eta^{n+2}).$$

Here, $\left(\frac{1}{\eta} S + J_i A_i + \xi \mu_i + \bar{u}_i \xi_i\right)$, which replaces $(S + J_i A_i + \xi \mu_i + \bar{u}_i \xi_i)$, is used to calculate Γ and η^n, where η is the loop number parameter. $o(\eta^{n+2})$ represents the high order correction with order higher than (n+2). However, how to define $\tilde{S}_n^0$ of each order n and how to determine the high order correction $o(\eta^{n+2})$ were not very clear in this method. This also was not clearly described in the paper of Julve and Tonin. This vagueness was later cleared up by Lee [7] who was able to determine the relation between $\tilde{S}_{n+1}^0$ and $\tilde{S}_n^0$. In his proof, it was clearly shown that, for each order of n, $\tilde{S}_n^0$ rigorously satisfies

$$\tilde{S}_n^0 * \tilde{S}_n^0 = 0$$

In this paper, we will modify and improve Lee's method. We will give a rigorous and clear proof that the gauge groups before and after renormalization are isomorphic in the presence of Abel subgroups, and that for each n, $\tilde{S}_n^0$ rigorously satisfies $\tilde{S}_n^0 * \tilde{S}_n^0 = 0$.

II.

In this section the Slavnov's identical equations in the presence of Abel subgroups and Higgs fields will be derived. The gauge group can be expressed as $G = A \otimes B$,

where A is not an Abel subgroup and B is an Abel subgroup. Without fermion field, the Lagrangians which are invariant under the gauge

transformation can be expressed as

$$\mathscr{L}_{inv}[A_\mu, B_\mu, S_\alpha] = \mathscr{L}_{inv}[A_\mu] + \mathscr{L}_{inv}[B_\mu] - \frac{1}{2}|D_\mu s|^2 - \lambda V(s^+ s),$$

$$\mathscr{L}_{inv}[A_\mu] = -\frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{abc} A_\mu^b A_\nu^c)^2,$$

$$\mathscr{L}_{inv}[B_\mu] = -\frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2,$$

$$(D_\mu s)^2 = \left|\left(\partial_\mu - \frac{i}{2} g \tilde{\tau}^a \cdot A_\mu^a - \frac{i}{2} g' B_\mu\right) s\right|^2. \qquad 2$$

where A_μ^a is the non-Abel gauge field, B_μ is the Abel gauge field, s_α and s_α^+ are the Higgs fields. We also choose

$$V(s^+ s) = (s_\alpha^+ s_\alpha)^2. \qquad 3$$

In the W-S model, for example, we have

$$s_1 = \frac{1}{\sqrt{2}}(\varphi_2 + i\varphi_1), \qquad s_2 = \frac{1}{\sqrt{2}}(\varphi_4 - i\varphi_3),$$

$$s_1^+ = \frac{1}{\sqrt{2}}(\varphi_2 - i\varphi_1), \qquad s_2^+ = \frac{1}{\sqrt{2}}(\varphi_4 + i\varphi_3),$$

$\tilde{\tau}^1$, $\tilde{\tau}^2$, $\tilde{\tau}^3$ are Pauli matrices.

The gauge fixed term of the R_ξ gauge is

$$-\frac{1}{2}\xi_A (C^{Aa})^2 - \frac{1}{2}\xi_B (C^B)^2, \qquad 4$$

where

$$C^{Aa} = \partial_\mu A_\mu^a + \frac{1}{\xi_A}\frac{gv}{2}\frac{i}{\sqrt{2}}(\delta_{1l}\tilde{\tau}^a_{lk} s_k - s_k^+ \tilde{\tau}^a_{kl}\delta_{l1}) \qquad (a = 1, 2, 3)$$

$$C^B = \partial_\mu B_\mu - \frac{1}{\xi_B}\frac{g' v}{2}\frac{i}{\sqrt{2}}(\delta_{1l}\tilde{\tau}^3_{lk} s_k - s_k^+ \tilde{\tau}^3_{kl}\delta_{l1}).$$

These expressions can be rewritten if the gauge is expanded into the general case of one gauge only (includes the R_ξ gauge)

$$C^{Aa} = F_i^{Aa} A_i + c_k^{Aa} s_k + c_k^{Aa+} s_k^+, \qquad 4'$$
$$C^B = F_\mu^B B_\mu + c_k^B s_k + c_k^{B+} s_k^+.$$

Since this is the more general case, it is not necessary to be confined in SU(2) and a is not necessary to be 1,2, or 3.

The gauge transformations are:

$$A_\mu^a \to A_\mu^{a'} = A_\mu^a - \frac{1}{g}(\partial_\mu \theta^a - g f_{abc}\theta^b A_\mu^c),$$

$$B_\mu \to B'_\mu = B_\mu - \frac{1}{g'}\partial_\mu \theta,$$

and

$$s_l \to s_l' = s_l + \left(-\frac{i}{2}\,\bar{\tau}^a_{lm}\theta^a - \frac{i}{2}\,\theta\delta_{lm}\right)s_m,$$

$$s_l^+ \to s_l^{+\prime} = s_l^+ + s_m^+\left(\theta^a\bar{\tau}^a_{ml}\frac{i}{2} + \delta_{ml}\theta\frac{i}{2}\right).$$

If $\theta^a = g\lambda^a$, $\theta = g'\lambda$, and the simplified functionals are used, then the above expressions can be rewritten as:

$$\begin{aligned} &A_i \to A_i' = A_i + (\Delta_i^{Ab} + g t^b_{ik} A_k)\lambda^b, \\ &B_\mu \to B_\mu' = B_\mu + \Delta_\mu^B \lambda, \\ &s_l \to s_l' = s_l - \frac{i}{2}\left(g\tau^a_{lm}\lambda^a s_m + g'\lambda s_l\right), \\ &s_l^+ \to s_l^{+\prime} = s_l^+ + \frac{i}{2}\left(g s_m^+ \tau^a_{ml}\lambda_a + g' s_l^+ \lambda\right). \end{aligned}$$ 5

Some symbols are simplified as below (α, β are the space-time indices; a,b,c are the group indices of the non-Abel subgroup; l, m are the group representation indices of the non-Abel subgroup).

$$A_i = A^a_\alpha, \qquad \Delta_i^{Ab} = -\partial_\alpha \delta^4(x_i - x_j)\delta_{ab},$$

where the i in the left hand side represents a,α, and x_i in the right hand side; the b in the left hand side represents b and x_j in the right hand side.

$$t^b_{ik} = f_{abc}\delta^4(x_i - x_k)\delta^4(x_i - x_j)\delta_{\alpha\beta},$$

where the i in the left hand side represents a,α, and x_i in the right hand side; the k in the left hand side represents c,β, and x_k in the right hand side; the b in the left hand side represents b and x_j in the right hand side.

Similar to the previous expressions, we have

$$\Delta_\mu^B = -\partial_\mu \delta^4(x_i - x_j), \quad \tau^a_{lm} = \bar{\tau}^a_{lm}\delta^4(x_i - x_k)\delta^4(x_i - x_j).$$

Same as in Ref.7, there are the following relations:

$$\begin{aligned} &[t^a, t^b] = f_{abc}t^c, \\ &t^a_{ik}\Delta_k^{Ab} - t^b_{ik}\Delta_k^{Aa} = f_{abc}\Delta_i^{Ac}, \\ &\left[-\frac{i}{2}\tau^a, -\frac{i}{2}\tau^b\right] = f_{abc}\left(-\frac{i}{2}\tau^c\right). \end{aligned}$$ 6

These relations do not appear in the Abel subgroups because the structure constant is zero. Now, from Eqs. (4) and (5) we have

$$-M^{a}_{\dot{a}b} = \frac{\delta c^{Aa}}{\delta\lambda^{b}} = F^{Aa}_{i}(\Delta^{Ab}_{i} + g t^{b}_{il} A_{l}) + c^{Aa}_{l}\left(-\frac{i}{2} g \tau^{b}_{lk} s_{k}\right) + c^{Aa+}_{l}\left(\frac{i}{2} g s^{+}_{k} \tau^{b}_{kl}\right),$$
$$-M^{a}_{\dot{a}} = \frac{\delta c^{Aa}}{\delta\lambda} = c^{Aa}_{l}\left(-\frac{i}{2} g' s_{l}\right) + c^{Aa+}_{l}\left(\frac{i}{2} g' s^{+}_{l}\right),$$
$$-M^{b}_{\dot{a}} = \frac{\delta c^{b}}{\delta\lambda} = F^{B}_{\mu}\Delta^{B}_{\mu} + c^{B}_{l}\left(-\frac{i}{2} g' s_{l}\right) + c^{B+}_{l}\left(\frac{i}{2} g' s^{+}_{l}\right),$$
$$-M^{B}_{\dot{\omega}} = \frac{\delta c^{b}}{\delta\lambda^{b}} = c^{B}_{l}\left(-\frac{i}{2} g \tau^{b}_{lk} s_{k}\right) + c^{B+}_{l}\left(\frac{i}{2} g s^{+}_{k}\tau^{b}_{kl}\right). \qquad 7$$

There are two sets of F-P fields needed to be chosen. One set is $\bar{u}_a$, u_n which is corresponding to the non-Abel subgroup; the other set is $\bar{u}$, u which is corresponding to Abel subgroup. The gauge compensated term should be

$$\mathcal{L}_{F-P} = \bar{u}_a M^{A}_{\omega b} u_b + \bar{u}_a M^{A}_{\omega} u + \bar{u} M^{B}_{\omega} u + \bar{u} M^{B}_{\omega b} u_b$$
$$= -\bar{u}_a F^{Aa}_{i}(\Delta^{Ab}_{i} + g t^{b}_{il} A_{l}) u_b$$
$$- c^{Aa}_{l}\bar{u}_a\left(-\frac{i}{2} g \tau^{b}_{lk} s_k\right) u_b - c^{Aa+}_{l}\bar{u}_a\left(\frac{i}{2} g s^{+}_{k}\tau^{b}_{kl}\right) u_b - \bar{u} F^{B}_{\mu}\Delta^{B}_{\mu} u$$
$$- c^{B}_{l}\bar{u}\left(-\frac{i}{2} g' s_l\right) u - c^{B+}_{l}\bar{u}\left(\frac{i}{2} g' s^{+}_{l}\right) u$$
$$- c^{Aa}_{l}\bar{u}_a\left(-\frac{i}{2} g' s_l\right) u - c^{Aa+}_{l}\bar{u}_a\left(\frac{i}{2} g' s^{+}_{l}\right) u$$
$$- c^{B}_{l}\bar{u}\left(-\frac{i}{2} g \tau^{b}_{lk} s_k\right) u_b - c^{B+}_{l}\bar{u}\left(\frac{i}{2} g s^{+}_{k}\tau^{b}_{k}\right) u_b. \qquad 8$$

Let's take

$$\mathcal{L} = \mathcal{L}_{inv}[A_i] + \mathcal{L}_{inv}[B_\mu] - \frac{1}{2}(Ds_a)^2 - \Lambda V(s^+ s)$$
$$- \frac{1}{2}\xi_A (C^{Aa})^2 - \frac{1}{2}\xi_B (C^{B})^2$$
$$+ (K^{A}_{i} - F^{Aa}_{i}\bar{u}_a)(\Delta^{Ab}_{i} + g t^{b}_{il} A_l) u_b + (K^{B}_{\mu} - F^{B}_{\mu}\bar{u})\Delta^{B}_{\mu} u$$
$$+ (K^{s}_{l} - c^{Aa}_{l}\bar{u}_a - c^{B}_{l}\bar{u})\left(-\frac{i}{2} g \tau^{b}_{lk} s_k u_b - \frac{i}{2} g' s_l u\right)$$
$$+ (K^{s+}_{l} - c^{Aa+}_{l}\bar{u}_a - c^{B+}_{l}\bar{u})\left(\frac{i}{2} g s^{+}_{k}\tau^{b}_{kl} u_b + \frac{i}{2} g' s^{+}_{l} u\right)$$
$$+ \frac{1}{2} g L_a f_{abc} u_b u_c, \qquad 9$$

where K^{A}_{i}, K^{B}_{μ}, K^{s}_{l}, K^{s+}_{l}, $\bar{u}_a$, u_a, $\bar{u}$, and u are mutually anticommuting. In this case, the B.R.S. transformation should be ($\delta\lambda$ is also anticommuting

with respect to K, ū,u,...., etc)

$$\delta A_i = (\Delta_i^{ab} + g t_{ij}^b A_j) u_b \delta\lambda, \qquad \delta B_\mu = (\Delta_\mu^B u)\delta\lambda,$$

$$\delta s_i = -\frac{i}{2}(g \tau_{ik}^b s_k u_b + g' s_i u)\delta\lambda,$$

$$\delta s_i^+ = \frac{i}{2}(g s_k^+ \tau_{ki}^b u_b + g' s_i^+ u)\delta\lambda,$$

$$\delta u_a = -\frac{1}{2} g f_{abc} u_b u_c \delta\lambda,$$

$$\delta \bar{u}_a = \xi_A (F_i^{Aa} A_i + c_k^{Aa} s_k + c_k^{Aa+} s_k^+)\delta\lambda,$$

$$\delta u = 0$$

$$\delta \bar{u} = \xi_B (F_\mu^B B_\mu + c_k^B s_k + c_k^{B+} s_k^+)\delta\lambda.$$

According to the calculation, the S^R in Eq.(9) is invariant under the B.R.S. transformation of Eq.(10). From this, the Slavnov identical equations which are satisfied by S^R can be derived

$$\frac{\delta \tilde{S}^R}{\delta K_i^A}\frac{\delta \tilde{S}^R}{\delta A_i} + \frac{\delta \tilde{S}^R}{\delta K_\mu^B}\frac{\delta \tilde{S}^R}{\delta B_\mu} + \frac{\delta \tilde{S}^R}{\delta K_i^\prime}\frac{\delta \tilde{S}^R}{\delta s_i} + \frac{\delta \tilde{S}^R}{\delta K_i^{\prime +}}\frac{\delta \tilde{S}^R}{\delta s_i^+} + \frac{\delta \tilde{S}^R}{\delta L_a}\frac{\delta \tilde{S}^R}{\delta u_a} = 0, \tag{11a}$$

$$-F_i^{Aa}\frac{\delta \tilde{S}^R}{\delta K_i^A} - c_i^{Aa}\frac{\delta \tilde{S}^R}{\delta K_i^\prime} - c_i^{Aa+}\frac{\delta \tilde{S}^R}{\delta K_i^{\prime +}} - \frac{\delta \tilde{S}^R}{\delta \bar{u}_a} = 0, \tag{11b}$$

$$-F_\mu^B\frac{\delta \tilde{S}^R}{\delta K_\mu^B} - c_i^B\frac{\delta \tilde{S}^R}{\delta K_i^\prime} - c_i^{B+}\frac{\delta \tilde{S}^R}{\delta K_i^{\prime +}} - \frac{\delta \tilde{S}^R}{\delta \bar{u}} = 0, \tag{11c}$$

$$\tilde{S}^R = S^R + \frac{1}{2}\xi_A (C^{Aa})^2 + \frac{1}{2}\xi_B (C^B)^2. \tag{11d}$$

S^R can be used to derive the (perturbation) vertex generating functional Γ. Since the integral volume element $d(A_i)d(B_\mu)d(\bar{u}_a)d(u_a)\cdot d(\bar{u})d(u)$ is invariant under the B.R.S. transformation, the Slavnov identical equations of Γ can be obtained

$$\frac{\delta \tilde{\Gamma}}{\delta K_i^A}\frac{\delta \tilde{\Gamma}}{\delta A_i} + \frac{\delta \tilde{\Gamma}}{\delta K_\mu^B}\frac{\delta \tilde{\Gamma}}{\delta B_\mu} + \frac{\delta \tilde{\Gamma}}{\delta K_i^\prime}\frac{\delta \tilde{\Gamma}}{\delta s_i} + \frac{\delta \tilde{\Gamma}}{\delta K_i^{\prime +}}\frac{\delta \tilde{\Gamma}}{\delta s_i^+} + \frac{\delta \tilde{\Gamma}}{\delta L_a}\frac{\delta \tilde{\Gamma}}{\delta u_a} = 0, \tag{12a}$$

$$-F_i^{Aa}\frac{\delta \tilde{\Gamma}}{\delta K_i^A} - c_i^{Aa}\frac{\delta \tilde{\Gamma}}{\delta K_i^\prime} - c_i^{Aa+}\frac{\delta \tilde{\Gamma}}{\delta K_i^{\prime +}} - \frac{\delta \tilde{\Gamma}}{\delta \bar{u}_a} = 0, \tag{12b}$$

$$-F_\mu^B\frac{\delta \tilde{\Gamma}}{\delta K_\mu^B} - c_i^B\frac{\delta \tilde{\Gamma}}{\delta K_i^\prime} - c_i^{B+}\frac{\delta \tilde{\Gamma}}{\delta K_i^{\prime +}} - \frac{\delta \tilde{\Gamma}}{\delta \bar{u}} = 0, \tag{12c}$$

$$\tilde{\Gamma} = \Gamma + \frac{1}{2}\xi_A (C^{Aa})^2 + \frac{1}{2}\xi_B (C^B)^2. \tag{12d}$$

Eqs.(11a) and (12a) can be simplified into

$$\tilde{S}^R * \tilde{S}^R = 0, \qquad \tilde{\Gamma} * \tilde{\Gamma} = 0.$$

III.

Corresponding to the Slavnov identical equations [illegible] and (12), the operat[illegible] defined by

$$\mathscr{G} = \mathscr{G}_0 + \mathscr{G}_1,$$

$$\mathscr{G}_0 = \frac{\delta \tilde{S}^R}{\delta K_i^A}\frac{\delta}{\delta A_i} + \frac{\delta \tilde{S}^R}{\delta K_\mu^B}\frac{\delta}{\delta B_\mu} + \frac{\delta \tilde{S}^R}{\delta K_i^l}\frac{\delta}{\delta s_l} + \frac{\delta \tilde{S}^R}{\delta K_i^{l+}}\frac{\delta}{\delta s_l^+} + \frac{\delta \tilde{S}^R}{\delta L_a}\frac{\delta}{\delta u_a}$$

$$= (\Delta_i^{ab} + g t_{ij}^b A_j) u_b \frac{\delta}{\delta A_i} + \Delta_\mu^a u \frac{\delta}{\delta B_\mu}$$

$$+ \left(-\frac{i}{2} g \tau_{lk}^b s_k u_b - \frac{i}{2} g' s_l u\right)\frac{\delta}{\delta s_l}$$

$$+ \left(\frac{i}{2} g s_k^+ \tau_{kl}^b u_b + \frac{i}{2} g' s_l^+ u\right)\frac{\delta}{\delta s_l^+} + \frac{1}{2} g f_{abc} u_b u_c \frac{\delta}{\delta u_a}, \qquad 13b$$

$$\mathscr{G}_1 = \frac{\delta \tilde{S}^R}{\delta A_i}\frac{\delta}{\delta K_i^A} + \frac{\delta \tilde{S}^R}{\delta B_\mu}\frac{\delta}{\delta K_\mu^B} + \frac{\delta \tilde{S}^R}{\delta s_l}\frac{\delta}{\delta K_i^l} + \frac{\delta \tilde{S}^R}{\delta s_l^+}\frac{\delta}{\delta K_i^{l+}} + \frac{\delta \tilde{S}^R}{\delta u_a}\frac{\delta}{\delta L_a}. \qquad 13c$$

They also satisfy the following relations

$$\mathscr{G}_0 \cdot \mathscr{G}_0 = 0, \qquad \mathscr{G}_0 \cdot \mathscr{G}_1 + \mathscr{G}_1 \cdot \mathscr{G}_0 = 0,$$

So,

$$\mathscr{G} \cdot \mathscr{G} = 0. \qquad 14$$

(Similar to the case of Ref.7, $\frac{\delta}{\delta u}$ does not appear in the above equations. The only difference is that some u terms appear in our expression but not in Ref.7. Since they always show up in the forms of $u_a u + u u_a$ (=0) and uu(=0), they can be eliminated.)

Now let's start our proof by the method of induction. First of all, we assume that n loops of renormalization has been done. The bare $\tilde{S}^o$ after n loops of renormalization is defined by $\tilde{S}_n^o$ (equal to S_n^B of Ref.6). It rigorously satisfies the following equations

$$\tilde{S}_n^o * \tilde{S}_n^o = 0,$$

$$-F_i^{Aa}\frac{\delta \tilde{S}_n^o}{\delta K_i^A} - c_l^{la}\frac{\delta \tilde{S}_n^o}{\delta K_i^l} - c_l^{la+}\frac{\delta \tilde{S}_n^o}{\delta K_i^{l+}} - \frac{\delta \tilde{S}_n^o}{\delta u_a} = 0,$$

$$-F_\mu^B\frac{\delta \tilde{S}_n^o}{\delta K_\mu^B} - c_l^B\frac{\delta \tilde{S}_n^o}{\delta K_i^l} - c_l^{B+}\frac{\delta \tilde{S}_n^o}{\delta K_i^{l+}} - \frac{\delta \tilde{S}_n^o}{\delta u} = 0. \qquad 15$$

The first equation means that $\tilde{S}_n^o$, after n loops of renormalization, is

invariant under the B.R.S. transformations (This is true for n=0, where $\tilde{S}^{o}_{n=0}=\tilde{S}^{R}$). After n loops of renormalization, the B.R.S. transformations are (the four K's and $\bar{u}_A$, $\bar{u}_B$ are all mutually anticommuting

$$\delta A_i = \frac{\delta S^0_n}{\delta K_i^A}\,\delta\lambda, \qquad \delta B_i = \frac{\delta S^0_n}{\delta K_i^B}\,\delta\lambda,$$

$$\delta\psi_i = \frac{\delta S^0_n}{\delta K_i}\,\delta\lambda, \qquad \delta\psi_i^{+} = \frac{\delta S^0_n}{\delta K_i^{+}}\,\delta\lambda,$$

$$\delta u_\alpha = -\frac{\delta S^0_n}{\delta L_\alpha}\,\delta\lambda, \qquad \delta \bar{u}_\alpha = \xi_A C^{A\alpha}\delta\lambda,$$

$$\delta u = 0, \qquad \delta\bar{u} = \xi_B C^{B}\delta\lambda. \qquad 16$$

Since S^o_n is invariant under the B.R.S. transformation of Eq.(16), the calculated $\tilde{\Gamma}$ by using S^o_n instead of S^R must satisfy the Slavnov identical equations. (There is one condition that the integral volume element must be invariant under the B.R.S. transformation of Eq. (16). It will be proved later that, after the S^o_n, S^o_{n+1}, ... etc. are determined, the sum of the Jacobian diagonal terms is still zero. So the integral volume element is invariant under the B.R.S transformation.) This implies that

$$\tilde{\Gamma} * \tilde{\Gamma} = 0,$$

$$-F_i^{A\alpha}\frac{\delta\tilde{\Gamma}}{\delta K_i^A} - c_i^{A\alpha}\frac{\delta\tilde{\Gamma}}{\delta K_i} - c_i^{A\alpha+}\frac{\delta\tilde{\Gamma}}{\delta K_i^{+}} - \frac{\delta\tilde{\Gamma}}{\delta u_\alpha} = 0,$$

$$-F_i^{B}\frac{\delta\tilde{\Gamma}}{\delta K_i^B} - c_i^{B}\frac{\delta\tilde{\Gamma}}{\delta K_i} - c_i^{B+}\frac{\delta\tilde{\Gamma}}{\delta K_i^{+}} - \frac{\delta\tilde{\Gamma}}{\delta \bar{u}} = 0. \qquad 17$$

Here $\tilde{\Gamma}$ can be expanded about the loop number (at first S^R is replaced by $\frac{1}{\hbar}S^R$):

$$\tilde{\Gamma} = \sum_{i=0}^{\infty} \hbar^i \tilde{\Gamma}_i. \qquad 18$$

According to the assumption of induction, $\tilde{\Gamma}_1$, $\tilde{\Gamma}_2$, ..., $\tilde{\Gamma}_n$ are all limited, and $\tilde{\Gamma}_{n+1}$ starts to become divergent. This divergence is a general divergence of n+1 loops. The divergences of all subloops (less than n+1 loops) have been eliminated. According to the discussion in Ref. 5, the divergent part of $\tilde{\Gamma}_{n+1}$ is expressed by

$\tilde{\Gamma}_{\mathrm{div}(n+1)}(S_n^0)$ and must satisfy:

$$\tilde{\Gamma}_{\mathrm{div}(n+1)}(S_n^0) * \tilde{S}^0 + \tilde{S}^0 * \tilde{\Gamma}_{\mathrm{div}(n+1)}(S_n^0) = (\mathcal{G}_0 + \mathcal{G}_1)\tilde{\Gamma}_{\mathrm{div}(n+1)}(S_n^0) = 0,$$
$$\left(-F_\mu^{A} \frac{\delta}{\delta K_\mu^A} - c_i^{A} \frac{\delta}{\delta K_i^{\prime}} - c_i^{A+} \frac{\delta}{\delta K_i^{\prime +}} - \frac{\delta}{\delta u_a}\right) \tilde{\Gamma}_{\mathrm{div}(n+1)}(S_n^0) = 0,$$
$$\left(-F_\mu^{b} \frac{\delta}{\delta K_\mu^B} - c_i^{B} \frac{\delta}{\delta K_i^{\prime}} - c_i^{B+} \frac{\delta}{\delta K_i^{\prime +}} - \frac{\delta}{\delta u}\right) \tilde{\Gamma}_{\mathrm{div}(n+1)}(S_n^0) = 0. \qquad (19)$$

In order to eliminate this divergence, we must find out the complete solutions of $\tilde{\Gamma}_{\mathrm{div}(n+1)}(S_n^0)$ which satisfies Eq. (19).

It should be noted that $\delta/\delta u$ does not show up both in the Slavnov identical equations and in $\mathcal{G}_0$ and $\mathcal{G}_1$. Therefore, there are three kinds of solutions. The first kind of solution is without F-P field; the second kind of solution has u_a and $\bar{u}_a$ (also can have u and $\bar{u}$); the third kind of solution has u but not $\bar{u}_a$. For the first two kinds of solutions, the proof of Jaglekar and Lee [8] is still applicable. In other words, the local complete solutions without F-P field and with u_a and $\bar{u}_a$ should have the form like:

$$G[A, B, s, s^+] + \mathcal{G}\mathcal{F}(A, B, s, s^+, u_a, \bar{u}_a, u, K, L). \qquad (20)$$

Although expression (20) may contain the third kind of solution (see F of Eq. (30)), it can not contain all the third kind of solutions. The complete solution of the third kind will be discussed later. Let's first study $G[A,B,s,s^+]$ which is a gauge invariant functional. Since $\tilde{\Gamma}$ is a local functional with dimension 4, and only $\mathcal{L}_{inv}[A_i]$, $\mathcal{L}_{inv}[B_\mu]$, $-\frac{1}{2}|Ds_a|^2$, and $V(s^+s)$ are the local gauge invariant functionals with dimension 4, so the general form of $G[A,B,s,s^+]$ should be like:

$$G[A,B,s,s^+] = \alpha(\epsilon)\mathcal{L}_{inv}[A_i] + \alpha'(\epsilon)\mathcal{L}_{inv}[B_\mu] - \alpha''(\epsilon)\left(\frac{1}{2}|Ds_a|^2\right) - \zeta(\epsilon)V(s^+s). \qquad (21)$$

(use the minimum renormalization, $\epsilon = n-4$)

For $\mathcal{G}\mathcal{F}$, $\mathcal{G}$ is the operator of Eq. (13a), and $\mathcal{F}$ must satisfy the following requirements:

(a) The F-P charge of $\mathcal{G}$ is +1, the F-P charge of $\tilde{\Gamma}$ is zero, so the F-P charge of $\mathcal{F}$ should be -1. (L_a:-2; K,$\bar{u}_a$,$\bar{u}$:-1; A,B,s:0; u_a,u:+1)

(b) The dimension of $\mathcal{G}$ is +1, the dimension of $\tilde{\Gamma}$ is 4, so the dimension of $\mathcal{F}$ should be 3 (K,L_a:+2; C^{Aa},C^{Aa+},C^B,C^{B+}:+1).

(c) All the indices of $\mathcal{G}$ are contracted, all the indices of $\tilde{\Gamma}$ are also contracted, so all the indices of $\mathcal{F}$ should be contracted.

From these three requirements, $\mathcal{F}$ only has seven terms $K_i^A A_i, K_\mu^B B_\mu, K_l^s s_l, K_l^{s+} s_l^+, K_l^s(d^b_{lm} c_m^{Ab+} + e c_l^{B+}),\ K_l^{s+}(d^{b+}_{ml} c_m^{Ab} + e^+ c_l^B), L_a u_a$, which contain K and L_a. No single term in $\mathcal{F}$ contains both K (or L_a) and u because of the violation of the above three requirements. $K_i^A t_{ij}^b t_{jk}^b A_k$ will come back to $K_i^A A_i$ because of $t_{ij}^b t_{jk}^b \sim C(G)\delta_{ik}$. $K_l^s \tau^b_{lm} \tau^b_{mk} s_k$ will come back to $K_l^s s_l$ because of $\tau^b_{lm}\tau^b_{mk} \sim C(R)\delta_{lk}$. Since the propagators satisfy $\overline{s_k^+ s_k^+} = 0$, $\overline{s_k s_k} = 0$, $\overline{s_k s_k^+} \neq 0$, both $K_l^s s_l^+$ and $K_l^{s+} s_l$ are excluded from $\mathcal{F}$.

(d) Due to the last two of the Slavnov equations (19), $K_i^A A_i$, $K_\mu^B B_\mu$, $K_l^s s_l$, and $K_l^{s+} s_l^+$ have to show up with $\bar{u}_a$ and $\bar{u}$ in the following forms:

$$(K_i^A - \bar{u}_a F_i^{Aa}) A_i, \tag{22}$$

$$(K_\mu^B - u F_\mu^B) B_\mu, \tag{23}$$

$$(K_l^s - c_l^{Aa} \bar{u}_a - c_l^B u) s_l, \tag{24}$$

$$(K_l^{s+} - c_l^{Aa+} \bar{u}_a - c_l^{B+} \bar{u}) s_l^+. \tag{25}$$

Besides, there are also some other terms which satisfy (a),(b),(c),(d) and contain K_l^s and K_l^{s+}. They are:

$$(K_l^s - c_l^{Aa} \bar{u}_a - c_l^B \bar{u})(d^b_{lm} c_m^{Ab+} + e c_l^{B+}), \tag{26}$$

$$(K_l^{s+} - c_l^{Aa+} \bar{u}_a - c_l^{B+} \bar{u})(d^{b+}_{ml} c_m^{Ab} + e^+ c_l^B). \tag{27}$$

It can be seen from the perturbation theory that d^{b+} and d^b are the group covariant constants, e^+ and e are constants. Since the

propagators satisfy $\overline{s_\ell s_\ell^+} \neq 0$, $\overline{s_\ell s_\ell} = \overline{s_\ell^+ s_\ell^+} = 0$, K_ℓ^s only multiplies with $c_m^{A\alpha +}$ and $c_\ell^{\beta +}$, and K_ℓ^{s+} only multiplies with $c_m^{A\alpha}$ and c_ℓ^{β}.

The term, which contains L_α, is not affected by requirement (d) and is still

$$L_\alpha u_\alpha . \qquad (\ )$$

If $\mathcal{G}$ is operated on (22), and (24) to (28), then as mentioned before only the second kind of solution (contains u_α and $\bar{u}_\alpha$ or u and $\bar{u}$) will be obtained.

If $\mathcal{G}$ is operated on (23), then the third kind of solution which contains B_μ will be obtained. But, as mentioned before, expression (20) does not contain all the third kind of solutions (i.e., for the Abel subgroup, the solution of Joglekar and Lee is not completed). Therefore, the other method must be used in order to find out the third kind of solutions.

If the perturbation is on S^R, $\tilde{\Gamma}$ must have the term of $\sim \bar{u} F_\mu^\beta \Delta_\mu^\beta u$ (i.e., the $\bar{u}u$ self-energy term). According to the third one of the Slavnov identical equations (19), this term of $\tilde{\Gamma}$ should have the form like

$$\sim (K_\mu^\beta - \bar{u} F_\mu^\beta)\, \Delta_\mu^\beta\, u$$

At the same time, if the perturbation is on $c_\ell^\beta \bar{u} s_\ell u$, $\tilde{\Gamma}$ must have the term of $\sim c_\ell^\beta \bar{u} s_\ell u$ (the $\bar{u} s_\ell u$ vertex of the renormalization). According to the last two of Eq. (19), this term should have the form like

$$\sim (K_l^s - c^{A\alpha}\bar{u}_\alpha - c_l^\beta \bar{u}) s_l u .$$

Similarly, $\tilde{\Gamma}$ should also have the term like

$$\sim (K_l^{s+} - c^{A\alpha +}\bar{u}_\alpha - c_l^{\beta +}\bar{u}) s_l^+ u .$$

If $\mathcal{G}$ is operated on all these terms, they will become

$$\mathcal{G}(K_\mu^B - \bar{u}F_\mu^B)\Delta_\mu^B u = \frac{\delta \mathcal{L}'_{inv}}{\delta B_\mu}\Delta_\mu^B u,$$

$$\mathcal{G}\left[(K_l^s - c_l^{Aa}\bar{u}_a - c_l^B\bar{u})\left(\frac{-i}{2}\right)g's_l u + (K_l^{s+} - c_l^{Aa+}\bar{u}_a - c_l^{B+}\bar{u})\left(\frac{i}{2}\right)g's_l^+ u\right]$$

$$= \frac{\delta \mathcal{L}'_{inv}}{\delta s_l}\left(\frac{-i}{2}g's_l u\right) + \frac{\delta \mathcal{L}'_{inv}}{\delta s_l^+}\left(\frac{i}{2}g's_l^+ u\right).$$

Since $\mathcal{L}'_{inv}$ is invariant under the $U(1)$ gauge transformation, it is zero if $\mathcal{G}$ is operated on

$$+(K_l^s - c_l^{Aa}\bar{u}_a - c_l^B\bar{u})\left(\frac{-i}{2}\right)g's_l u + (K_l^{s+} - c_l^{Aa+}\bar{u}_a - c_l^{B+}\bar{u})\left(\frac{i}{2}\right)g's_l^+ u \qquad 29$$

Therefore, Eq. (29) is also a solution and should be the third kind of solution. However, it can not be written as in the form of (20).

It also should be mentioned that $\mathcal{G}$x(23) is the only third kind of solution which contains B_μ, and that (29) is the only third kind of solution which does not contains B_μ. For the former case, since only $\mathcal{G}\frac{\delta \mathcal{L}'_{inv}}{\delta B_\mu}B_\mu = \frac{\delta \mathcal{L}'_{inv}}{\delta B_\mu}\Delta_\mu^B u$ can cancel out

$$\mathcal{G}(K_\mu^B - \bar{u}F_\mu)\Delta_\mu^B u = \frac{\delta \mathcal{L}'_{inv}}{\delta B_\mu}\Delta_\mu^B u$$

, but not the other functional of B_μ.

$$\mathcal{G}\times(23) = \frac{\delta \mathcal{L}'_{inv}}{\delta B_\mu}B_\mu - (K_\mu^B - \bar{u}F_\mu^B)\Delta_\mu^B u$$

is the only third kind of solution* which contains B_μ. For the latter case, since the F-P charge of u is +1, when it shows up in $\tilde{\Gamma}$ it must be the combination of $K_\mu^B u$, $K_l^s u$, $K_l^{s+} u$, $\bar{u}u$, $\bar{u}_a u$, and $L_a u_a u$. The dimension of $\tilde{\Gamma}$ is 4. Although the dimension of $L_a u_a u$ is also 4, it should be excluded. This is because the form of $L_a u_a$.... can not be obtained from the perturbation on $L_a f_{abc} u_b u_c$. $\bar{u}u\bar{u}u$ and $\bar{u}_a u\bar{u}_a u$ which have dimension 4 also have to be excluded. This is because uu=0 and $\bar{u}_a \bar{u}_a=0$. In fact, according to the last two of equations (19), $\bar{u}_a u_b \bar{u}u$ only can show up in the forms of $(K_l^s - c_l^{Aa}\bar{u}_a - c_l^B\bar{u})u_a$, $(K_l^{s+} - c_l^{Aa+}\bar{u}_a - c_l^{B+}\bar{u})u_a$, or $(K_i^A - \bar{u}_a F_i^{Aa})u_b$ multiplying $(K_l^s - c_l^{Aa}\bar{u}_a - c_l^B\bar{u})u$, $(K_l^{s+} - c_l^{Aa+}\bar{u}_a - c_l^{B+}\bar{u})u$, or

* see F of Eq. (30)

$(K_i^A - \bar{u}_a F_i^{Aa})u$. Because of this, the dimension will be more than 4. So they should be excluded (here there is no perturbation for the vacuum spontaneous symmetry breaking). Therefore the solutions, which satisfy the last two of the Slavnov equations (19), have F-P charge of zero and dimension of 4, are consistent with the perturbation, have all their indices contracted, and do not contain B_μ and u_a but contain u, only can have the following three combinations (A_i has a different index and can not be included):

$$\sim (K_\mu^B - \bar{u} F_\mu^B)\Delta_\mu^B u,$$
$$\sim (K_l^s - c_l^{Aa}\bar{u}_a - c_l^B \bar{u}) s_l u,$$
$$\sim (K_l^{s+} - c_l^{Aa+} u_a - c_l^{B+}\bar{u}) s_l^+ u.$$

(The reason why $K_l^s s_l^+$ and $K_l^{s+} s_l$ do not show up here has been discussed before.) If the three terms above also want to satisfy the first one of the Slavnov equations (19), the only possibility is that these three terms must combine themselves into the form of (29). So, (29) is the only third kind of solution without containing B_μ.

IV.

All the independent solutions which satisfy the requirements of $\tilde{\Gamma}^{\text{div}}_{(n+1)}(S_n^o)$ have been found. They are (from (21) to (29)):

$$A = \mathscr{L}_{\text{inv}}[A_i], \qquad \underset{\bullet}{B} = \mathscr{L}_{\text{inv}}[B_\mu],$$

$$C = -\frac{1}{2}|Ds_a|^2, \qquad D = V(s^+ s),$$

$$E = \mathscr{G}(K_i^A - \bar{u}_a F_i^{Aa}) A_i$$
$$= \frac{\delta \mathscr{L}'_{\text{inv}}}{\delta A_i} A_i - (K_i^A - \bar{u}_a F_i^{Aa})\Delta_i^{Ab} u_b,$$

$$F = \mathscr{G}(K_\mu^B - \bar{u} F_\mu^B) B_\mu = \frac{\delta \mathscr{L}'_{\text{inv}}}{\delta B_\mu} B_\mu - (K_\mu^B - \bar{u} F_\mu^B)\Delta_\mu^B u,$$

$$G = \mathscr{G}(K_l^s - c_l^{Aa}\bar{u}_a - c_l^B \bar{u}) s_l$$
$$= \frac{\delta \tilde{S}^R}{\delta s_l} s_l - (K_l^s - c_l^{Aa}\bar{u}_a - c_l^B \bar{u})\left(-\frac{i}{2} g \tau_{lk}^b s_k u_b - \frac{i}{2} g' s_l u\right),$$

$$H = \mathscr{G}(K_l^{s+} - c_l^{Aa+}\bar{u}_a - c_l^{B+}\bar{u}) s_l^+$$
$$= \frac{\delta \tilde{S}^R}{\delta s_l^+} s_l^+ - (K_l^{s+} - c_l^{Aa+}\bar{u}_a - c_l^{B+}\bar{u})\left(\frac{i}{2} g s_k^+ \tau_{kl}^b u_b + \frac{i}{2} g' s_l^+ u\right),$$

$$I = \mathscr{G}(K_l^s - c_l^{Aa}\bar{u}_a - c_l^B \bar{u})(d_{lm}^b c_m^{Ab+} + e c_l^{B+})$$

$$= \frac{\delta \tilde{S}^{\kappa}}{\delta s_l} (d^{b}_{lm} c^{Ab+}_{m} + e c^{B+}_{l}),$$

$$J = \mathcal{G}(K^{+}_{l} - c^{Aa+}_{l} \bar{u}_{a} - c^{B+}_{l} \bar{u})(d^{b+}_{ml} c^{Ab}_{m} + e^{+} c^{B}_{l})$$

$$= \frac{\delta \tilde{S}^{\kappa}}{\delta s^{+}_{l}} (d^{b+}_{ml} c^{Ab}_{m} + e^{+} c^{B}_{l}),$$

$$K = -\mathcal{G}(L_a u_a) = -\frac{g}{2} L_a \epsilon_{abc} u_b u_c - \frac{\delta \tilde{S}^{\kappa}}{\delta u_a} u_a,$$

$$L = u \frac{\delta \tilde{S}^{\kappa}}{\delta u}. \qquad 30$$

For convenience, these twelve independent solutions are rearranged. They become:

$$A - K = \frac{A_i}{2} \frac{\delta \tilde{S}^{\kappa}}{\delta A_i} - \frac{g}{2} \frac{\partial \tilde{S}^{\kappa}}{\partial g} + \frac{L_a}{2} \frac{\delta \tilde{S}^{\kappa}}{\delta L_a} - \frac{1}{2} \left(K^{A}_{i} \frac{\delta}{\delta K^{A}_{i}} + u_a \frac{\delta}{\delta u_a} + \bar{u}_a \frac{\delta}{\delta \bar{u}_a} \right) \tilde{S}^{\kappa}$$

$$+ \frac{1}{2} c^{Aa}_{l} \frac{\delta \tilde{S}^{\kappa}}{\delta c^{Aa}_{l}} + \frac{1}{2} c^{Aa+}_{l} \frac{\delta \tilde{S}^{\kappa}}{\delta c^{Aa+}_{l}}. \qquad 31a$$

(It will be zero if $\frac{A_i}{2} \frac{\delta}{\delta A_i} - \frac{g}{2} \frac{\partial}{\partial g}$ operates on the terms containing gA_i)

$$B - L = \frac{B_\mu}{2} \frac{\delta \tilde{S}^{\kappa}}{\delta B_\mu} - \frac{g'}{2} \frac{\partial \tilde{S}^{\kappa}}{\partial g'} - \frac{1}{2} \left(K^{B}_{\mu} \frac{\delta}{\delta K^{B}_{\mu}} + u \frac{\delta}{\delta u} + \bar{u} \frac{\delta}{\delta \bar{u}} \right) \tilde{S}^{\kappa}$$

$$+ \frac{1}{2} c^{B}_{l} \frac{\delta \tilde{S}^{\kappa}}{\delta c^{B}_{l}} + \frac{1}{2} c^{B+}_{l} \frac{\delta \tilde{S}^{\kappa}}{\delta c^{B+}_{l}}. \qquad 31b$$

(For all those terms containing $g'B_\mu$ will become zero if they are operated by $\frac{B_\mu}{2} \frac{\delta}{\delta B_\mu} - \frac{g'}{2} \frac{\partial}{\partial g'}$)

$$C + 2D = \frac{s_a}{2} \frac{\delta \tilde{S}^{\kappa}}{\delta s_a} + \frac{s^{+}_{a}}{2} \frac{\delta \tilde{S}^{\kappa}}{\delta s^{+}_{a}} - \frac{1}{2} \left(K_l \frac{\delta}{\delta K_l} + c^{Aa}_{l} \frac{\delta}{\delta c^{Aa}_{l}} + c^{B}_{l} \frac{\delta}{\delta c^{B}_{l}} \right) \tilde{S}^{\kappa} \qquad 31c$$

$$- \frac{1}{2} \left(K^{+}_{l} \frac{\delta}{\delta K^{+}_{l}} + c^{Aa+}_{l} \frac{\delta}{\delta c^{Aa+}_{l}} + c^{B+}_{l} \frac{\delta}{\delta c^{B+}} \right) \tilde{S}^{\kappa}, \qquad 31d$$

$$D = V(s^{+}s),$$

$$-E + K = -A_i \frac{\delta \tilde{S}^{\kappa}}{\delta A_i} + \left(K^{A}_{i} \frac{\delta}{\delta K^{A}_{i}} + u_a \frac{\delta}{\delta u_a} + \bar{u}_a \frac{\delta}{\delta \bar{u}_a} \right) \tilde{S}^{\kappa} \qquad 31e$$

$$- \left(c^{Aa}_{l} \frac{\delta}{\delta c^{Aa}_{l}} + c^{Aa+}_{l} \frac{\delta}{\delta c^{Aa+}_{l}} \right) \tilde{S}^{\kappa} - L_a \frac{\delta \tilde{S}^{\kappa}}{\delta L_a},$$

$$-F + L = -B_\mu \frac{\delta \tilde{S}^{\kappa}}{\delta B_\mu} + \left(K^{B}_{\mu} \frac{\delta}{\delta K^{B}_{\mu}} + u \frac{\delta}{\delta u} + \bar{u} \frac{\delta}{\delta \bar{u}} \right) \tilde{S}^{\kappa} \qquad 31f$$

$$- \left(c^{B}_{l} \frac{\delta}{\delta c^{B}_{l}} + c^{B+}_{l} \frac{\delta}{\delta c^{B+}_{l}} \right) \tilde{S}^{\kappa}, \qquad 31g$$

$$-G = \left(K_l \frac{\delta}{\delta K_l} + c^{Aa}_{l} \frac{\delta}{\delta c^{Aa}_{l}} + c^{B}_{l} \frac{\delta}{\delta c^{B}_{l}} - s_l \frac{\delta}{\delta s_l} \right) \tilde{S}^{\kappa},$$

$$-H = \left(K^{+}_{l} \frac{\delta}{\delta K^{+}_{l}} + c^{Aa+}_{l} \frac{\delta}{\delta c^{Aa+}_{l}} + c^{B+}_{l} \frac{\delta}{\delta c^{B+}_{l}} - s^{+}_{l} \frac{\delta}{\delta s^{+}_{l}} \right) \tilde{S}^{\kappa}, \qquad 31h$$

$$I = (d^{b}_{lm} c^{Ab+}_{m} + e c^{B+}_{l}) \frac{\delta \tilde{S}^{\kappa}}{\delta s_l}, \qquad 31i$$

$$J = (d^{\mu +}_{mi} c^{\mu}_{m} + \epsilon^{+} c^{B}_{l}) \frac{\delta \tilde{S}^{R}}{\delta s^{+}_{l}}, \tag{31j}$$

$$K + L = \frac{1}{2}\Big(K^{A}_{i} \frac{\delta}{\delta K^{A}_{i}} + u_{a} \frac{\delta}{\delta u_{a}} + \bar{u}_{a} \frac{\delta}{\delta \bar{u}_{a}} + K^{B}_{\mu} \frac{\delta}{\delta K^{B}_{\mu}} + u \frac{\delta}{\delta u} + \bar{u} \frac{\delta}{\delta \bar{u}} \tag{31k}$$

$$+ K'_{l} \frac{\delta}{\delta K'_{l}} + K'^{+}_{l} \frac{\delta}{\delta K'^{+}_{l}} \Big) \tilde{S}^{R},$$

$$L = u \frac{\delta \tilde{S}^{R}}{\delta u}. \tag{31l}$$

So, the general form of $\bar{\Gamma}^{\text{div}}_{(n+1)}(S^{0}_{n})$ is

$$\begin{aligned}
\bar{\Gamma}^{\text{div}}_{(n+1)}(S^{0}_{n}) = {} & \alpha(e)(31a) + \alpha'(e)(31b) + \alpha''(e)(31c) + \zeta(e)(31d) \\
& + \beta(e)(31e) + \beta'(e)(31f) + \beta''_{1}(e)(31g) \\
& + \beta''_{2}(e)(31h) + \delta_{1}(e)(31i) + \delta_{2}(e)(31j) \\
& - \gamma(e)(31k) + \eta(e)(31l)
\end{aligned}$$

$$\begin{aligned}
\Longrightarrow \Big[& \Big(\frac{\alpha(e)}{2} - \beta(e) \Big)\Big(A_{i} \frac{\delta}{\delta A_{i}} + L_{a} \frac{\delta}{\delta L_{a}} \Big) + \Big(\frac{\alpha'(e)}{2} - \beta'(e) \Big) B_{\mu} \frac{\delta}{\delta B_{\mu}} \\
& + \Big(\frac{\alpha''(e)}{2} - \beta''(e) \Big)\Big(s_{l} \frac{\delta}{\delta s_{l}} + s^{+}_{l} \frac{\delta}{\delta s^{+}_{l}} \Big) - \frac{\alpha(e)}{2} g \frac{\partial}{\partial g} - \frac{\alpha'(e)}{2} g' \frac{\partial}{\partial g'} \\
& + \Big(-\frac{\alpha(e)}{2} + \beta(e) - \frac{\gamma(e)}{2} \Big)\Big(K^{A}_{i} \frac{\delta}{\delta K^{A}_{i}} + u_{a} \frac{\delta}{\delta u_{a}} + \bar{u}_{a} \frac{\delta}{\delta \bar{u}_{a}} \Big) \\
& + \Big(-\frac{\alpha'(e)}{2} + \beta'(e) - \frac{\gamma(e)}{2} \Big)\Big(K^{B}_{\mu} \frac{\delta}{\delta K^{B}_{\mu}} + u \frac{\delta}{\delta u} + \bar{u} \frac{\delta}{\delta \bar{u}} \Big) \\
& + \Big(-\Big(\frac{\alpha''(e)}{2} - \beta''(e) \Big) + \Big(-\frac{\alpha(e)}{2} + \beta(e) - \frac{\gamma(e)}{2} \Big) \\
& + \Big(\frac{\alpha(e)}{2} - \beta(e) \Big) \Big) \cdot \Big(K'_{l} \frac{\delta}{\delta K'_{l}} + K'^{+}_{l} \frac{\delta}{\delta K'^{+}_{l}} \Big) \\
& + \Big(\Big(\frac{\alpha(e)}{2} - \beta(e) \Big) - \Big(\frac{\alpha''(e)}{2} - \beta''(e) \Big) \Big) \cdot \Big(c^{\mu a}_{l} \frac{\delta}{\delta c^{\mu a}_{l}} + c^{\mu a +}_{l} \frac{\delta}{\delta c^{\mu a +}_{l}} \Big) \\
& + \Big(\Big(\frac{\alpha'(e)}{2} - \beta'(e) \Big) - \Big(\frac{\alpha''(e)}{2} - \beta''(e) \Big) \Big) \cdot \Big(c^{B}_{l} \frac{\delta}{\delta c^{B}_{l}} + c^{B+}_{l} \frac{\delta}{\delta c^{B+}_{l}} \Big) \\
& - v_{l(n+1)} \frac{\delta}{\delta s_{l}} - v^{+}_{l(n+1)} \frac{\delta}{\delta s^{+}_{l}} \Big] \tilde{S}^{R} + \zeta(e) V(s^{+}s).
\end{aligned} \tag{32}$$

Some points should be explained. They are:

(a) Since u and $\bar{u}$ have common renormalization constant $\bar{Z}'_{3}$, so let $\gamma(e) = 0$.

(b) Since s_{l} and s^{+}_{l} have common renormalization constant Z_{s} (i.e., the Z_{s} of $\overline{s_{l}\, s^{+}_{l}}$ propagator), so let

$$\beta''_{1}(e) = \beta''_{2}(e) = \beta''_{3}(e).$$

(c) $\zeta(e)V(s^{+}s)$ can be eliminated by the renormalization of Λ in

Eq. (2) (For example, we can choose $\Lambda_{n+1} = \Lambda_n + \eta^{n+1}\zeta(x))$). There is an independent Z_4.

$$v_{l(n+1)} = -(d^{a}_{lm} c^{Ab+}_{m} + e c^{B+}_{l})\theta_1(x),$$
$$v^{+}_{l(n+1)} = -(d^{a-}_{ml} c^{Ab}_{m} + e^{+} c^{B}_{l})\theta_1(x).$$ 33a

Let

$$s^{0}_{l(n+1)} = (Z_s)^{1/2}_{n+1}(\bar{s}_l + (v_l)_{n+1}),$$
$$s^{0}_{l(n)} = (Z_s)^{1/2}_{n}(\bar{s}_l + (v_l)_{n}),$$ 33b

where

$$(Z_s)_i = 1 + \eta z_{s(1)} + \eta^2 z_{s(2)} + \cdots + \eta^i z_{s(i)},$$
$$(v_l)_i = v_{l(0)} + \eta v_{l(1)} + \eta^2 v_{l(2)} + \cdots + \eta^i v_{l(i)}.$$ 33c

Here the zeroth order does not have spontaneous breaking, and $v_{l(0)}=0$. $v_{l(1)}, \cdots, v_{l(n)}, v_{l(n+1)}$ are gauge related (related to c^{Ab}_{l}, c^{B}_{l}, c^{Ab+}_{l}, c^{B+}_{l}) cancelling terms of the polliwog diagram. They can be eliminated by renormalization as following:

$$\begin{aligned} s^{0}_{l(n+1)} - s^{0}_{l(n)} &= ((Z_s)^{1/2}_{n+1}(\bar{s}_l + (v_l)_{n+1}) - (Z_s)^{1/2}_{n}(\bar{s}_l + (v_l)_{n})) \\ &= \left(\left(1 + \frac{\eta^{n+1} z_{s(n+1)}}{1 + \eta z_{s(1)} + \cdots \eta^{n} z_{s(n)}}\right)^{1/2} - 1\right)(Z_s)^{1/2}_{n}\bar{s}_l \\ &\quad + \left(\left(1 + \frac{\eta^{n+1} z_{s(n+1)}}{1 + \eta z_{s(1)} + \cdots + \eta^{n} z_{s(n)}}\right)^{1/2}(\eta v_{l(1)} + \cdots \right. \\ &\quad \left. + \eta^{n+1} v_{l(n+1)}) - (\eta v_{l(1)} + \cdots + \eta^{n} v_{l(n)})\right)(Z_s)^{1/2}_{n} \\ &= \frac{1}{2}\eta^{n+1} z_{s(n+1)}\bar{s}_l + \eta^{n+1} v_{l(n+1)} + o(\eta^{n+2}). \end{aligned}$$ 34a

Similarly, we have

$$s^{+0}_{l(n+1)} - s^{+0}_{l(n)} = \frac{1}{2}\eta^{n+1} z_{s(n+1)}\bar{s}^{+}_{l} + \eta^{n+1} v^{+}_{l(n+1)} + o(\eta^{n+2}).$$ 34b

Again, let

$$A^{0}_{i(n)} = (Z_3)^{1/2}_{n} A_i, \quad u^{0}_{a(n)} = (\tilde{Z}_3)^{1/2}_{n} u_a, \quad \bar{u}^{0}_{a(n)} = (\tilde{Z}_3)^{1/2}_{n} \bar{u}_a,$$
$$B^{0}_{\mu(n)} = (Z'_3)^{1/2} B_\mu, \quad u^{0}_{(n)} = (\tilde{Z}'_3)^{1/2}_{n} u, \quad \bar{u}^{0}_{(n)} = (\tilde{Z}'_3)^{1/2}_{n} \bar{u}.$$ 34c

If every $(Z)_n$ is expanded by

$$(Z)_n = 1 + \eta z_{(1)} + \cdots + \eta^n z_{(n)}.$$ 34d

then we have

$$A^{0}_{i(n+1)} - A^{0}_{i(n)} = \frac{1}{2}\eta^{n+1} z_{3(n+1)} A_i + o(\eta^{n+2})$$

We also can [illegible] neglect ( )$_n$ symbol)

$$g^0 = \frac{\tilde{Z}_1}{\tilde{Z}_3 Z_3^{1/2}} g, \quad g'^0 = \frac{\tilde{Z}'_1}{\tilde{Z}_3 Z_3'^{1/2}} g', \quad \xi_A^0 = \frac{\xi_A}{Z_3}, \quad \xi_B^0 = \frac{\xi_B}{Z_3},$$

$$K_\mu^{A0} = (\tilde{Z}_3)^{1/2} K_\mu^A, \quad K_\mu^{B0} = (\tilde{Z}'_3)^{1/2} K_\mu^B, \quad L_\mu^0 = (Z_3)^{1/2} L_\mu,$$

$$K_i^{A0} = \frac{\tilde{Z}_3^{1/2} Z_3^{1/2}}{Z_i^{1/2}} K_i^A = \frac{\tilde{Z}_3'^{1/2} Z_3'^{1/2}}{Z_i^{1/2}} K_i^B,$$

$$K_i^{A+0} = \frac{\tilde{Z}_3^{1/2} Z_3^{1/2}}{Z_i^{1/2}}, \quad K_i^{B+0} = \frac{\tilde{Z}_3'^{1/2} Z_3'^{1/2}}{Z_i^{1/2}} K_i^{B+},$$

$$c_i^{A0} = \frac{Z_3^{1/2}}{Z_i^{1/2}} c_i^A, \quad c_i^{B0} = \frac{Z_3'^{1/2}}{Z_i^{1/2}} c_i^B,$$

$$c_i^{A+0} = \frac{Z_3^{1/2}}{Z_i^{1/2}} c_i^{A+}, \quad c_i^{B+0} = \frac{Z_3'^{1/2}}{Z_i^{1/2}} c_i^{B+}. \tag{34e}$$

Compar[illegible] 32), then the $z_{(n+1)}$'s t[illegible]

$$\frac{\alpha(\epsilon)}{2} - \beta(\epsilon) = -\frac{z_{3(n+1)}}{2}, \quad -\frac{\alpha(\epsilon)}{2} + \beta(\epsilon) - \frac{\gamma(\epsilon)}{2} = -\frac{\tilde{z}_{3(n+1)}}{2},$$

$$\frac{\alpha'(\epsilon)}{2} - \beta'(\epsilon) = -\frac{z'_{3(n+1)}}{2}, \quad -\frac{\alpha'(\epsilon)}{2} + \beta'(\epsilon) - \frac{\gamma(\epsilon)}{2} = -\frac{\tilde{z}'_{3(n+1)}}{2},$$

$$\frac{\alpha''(\epsilon)}{2} - \beta''(\epsilon) = -\frac{z_{3(n+1)}}{2},$$

$$-\frac{\alpha(\epsilon)}{2} = -\left(\tilde{z}_{1(n+1)} - \tilde{z}_{3(n+1)} - \frac{1}{2} z_{3(n+1)}\right),$$

$$-\frac{\alpha'(\epsilon)}{2} = -\left(\tilde{z}'_{1(n+1)} - \tilde{z}'_{3(n+1)} - \frac{1}{2} z'_{3(n+1)}\right). \tag{35}$$

If the A,B,s,s^r,K,L,c,g,g',ξ,...,$\bar{u}$, and u of S^R are replaced by $A^0_{(n+1)}$, $B^0_{(n+1)}$, $s^0_{(n+1)}$, $s^{r0}_{(n+1)}$, $K^0_{(n+1)}$, ...$\bar{u}^0_{(n+1)}$, and $u^0_{(n+1)}$, then S^R is redefined as S^0_{n+1}. Similarly, if they are replaced by $A^0_{(n)}$, $B^0_{(n)}$, $s^0_{(n)}$, $s^{+0}_{(n)}$, $K^0_{(n)}$, ...$\bar{u}^0_{(n)}$, and $u^0_{(n)}$, then S^R is redefined as S^0_n. After these replacements, we have

$$\bar{\Gamma}_{(n+1)}(S^0_{n+1}) = \bar{\Gamma}_{(n+1)}(S^0_n) + (\tilde{S}^0_{n+1} - \tilde{S}^0_n) + o(\eta^{n+1}). \tag{36}$$

Because of Eqs. (32) and (35), $(\tilde{S}^0_{n+1} - \tilde{S}^0_n)$ $(\sim o(\eta^{n+1}))$ will cancel out all the divergent terms $\eta^{n+1}\tilde{\Gamma}^{div}_{(n+1)}(s^0_n)$ $(\sim o(\eta^{n+1}))$; this is a general divergence of (n+1) order) of $\tilde{\Gamma}_{(n+1)}(s^0_n)$. But this will not affect those terms with an order less than η^{n+1}. So, for $\tilde{\Gamma}_{(n+1)}(s^0_{n+1})$, not only those limited terms η', η^2, ..., η^n are still limited, η^{n+1} also becomes limited. This is the result we want for the renormalization.

Since s^{0}_{n+1} and s^{0}_{n} are obtained from the variant transformation of S^{R}, the Slavnov identical equations should hold after the variant transforma[illegible]

$$\frac{\delta \tilde{S}^{0}_{n+1}}{\delta K^{A0}_{i(n+1)}} \frac{\delta \tilde{S}^{0}_{n+1}}{\delta A^{0}_{i(n+1)}} + \frac{\delta \tilde{S}^{0}_{n+1}}{\delta K^{B}_{\mu(n+1)}} \frac{\delta \tilde{S}^{0}_{n+1}}{\delta B^{0}_{\mu(n+1)}} + \frac{\delta \tilde{S}^{0}_{n+1}}{\delta K^{0}_{i(n+1)}} \frac{\delta \tilde{S}^{0}_{n+1}}{\delta \phi^{0}_{i(n+1)}}$$
$$+ \frac{\delta \tilde{S}^{0}_{n+1}}{\delta K^{\prime 0}_{i(n+1)}} \frac{\delta \tilde{S}^{0}_{n+1}}{\delta \phi^{\prime}_{i(n+1)}} + \frac{\delta \tilde{S}^{0}_{n+1}}{\delta L^{0}_{\alpha(n+1)}} \cdot \frac{\delta \tilde{S}^{0}_{n+1}}{\delta u^{0}_{\alpha(n+1)}} = 0,$$
$$- F^{A\alpha}_{i} \frac{\delta \tilde{S}^{0}_{n+1}}{\delta K^{A\alpha}_{i(n+1)}} - c^{A\alpha}_{i(n+1)} \frac{\delta \tilde{S}^{0}_{n+1}}{\delta K^{\prime}_{i(n+1)}} - c^{A\alpha+0}_{i(n+1)} \frac{\delta \tilde{S}^{0}_{n+1}}{\delta K^{\prime +0}_{i(n+1)}} - \frac{\delta \tilde{S}^{0}_{n+1}}{\delta u^{0}_{\alpha(n+1)}} = 0,$$
$$- F^{B}_{\mu} \frac{\delta \tilde{S}^{0}_{n+1}}{\delta K^{B0}_{\mu(n+1)}} - c^{B0}_{i(n+1)} \frac{\delta \tilde{S}^{0}_{n+1}}{\delta K^{\prime 0}_{i(n+1)}} - c^{B+0}_{i(n+1)} \frac{\delta \tilde{S}^{0}_{n+1}}{\delta K^{\prime +0}_{i(n+1)}} - \frac{\delta \tilde{S}^{0}_{n+1}}{\delta u^{0}_{(n+1)}} = 0, \qquad 37$$
$$\tilde{S}^{0}_{n+1} = s^{0}_{n+1} + \frac{1}{2} \xi^{0}_{A(n+1)} (C^{A\alpha})^{2}_{n+1} + \frac{1}{2} \xi^{0}_{B(n+1)} (C^{B\nu})^{2}_{n+1}.$$

From the multiplication relation (Eq. 34) of the renormalization, we have (see expression [illegible])

$$\frac{1}{2} \xi^{0}_{A} (C^{A\alpha})^{2} + \frac{1}{2} \xi^{0}_{B} (C^{B\nu})^{2} = \frac{1}{2} \xi_{A} (C^{A\alpha})^{2} + \frac{1}{2} \xi_{B} (C^{B})^{2}.$$

So the gauge fixed term is invariant under renormalization. Because of the following equation (from Eq. 35)

$$\frac{1}{2} z_{3(n+1)} + \frac{1}{2} \tilde{z}_{3(n+1)} = \frac{1}{2} z^{\prime}_{3(n+1)} + \frac{1}{2} \tilde{z}^{\prime}_{3(n+1)} = \frac{\gamma(e)}{2}.$$

we have (the equation above is true for each *order*)

$$Z_3 \tilde{Z}_3 = Z^{\prime}_3 Z^{\prime}_3.$$

So, after eliminating the common constant factor, Eq. (37) can be rewritten as

$$\frac{\delta \tilde{S}^{0}_{n+1}}{\delta K^{A}_{i}} \frac{\delta \tilde{S}^{0}_{n+1}}{\delta A_{i}} + \frac{\delta \tilde{S}^{0}_{n+1}}{\delta K^{B}_{\mu}} \frac{\delta \tilde{S}^{0}}{\delta B_{\mu}} + \frac{\delta \tilde{S}^{0}_{n+1}}{\delta K^{\prime}_{i}} \frac{\delta \tilde{S}^{0}_{n+1}}{\delta \phi_{i}} + \frac{\delta \tilde{S}^{0}_{n+1}}{\delta K^{\prime +}_{i}} \frac{\delta \tilde{S}^{0}_{n+1}}{\delta \phi^{+}_{i}} + \frac{\delta \tilde{S}^{0}_{n+1}}{\delta L_{\alpha}} \frac{\delta \tilde{S}^{0}_{n+1}}{\delta u_{\alpha}} = 0,$$
$$- F^{A\alpha}_{i} \frac{\delta \tilde{S}^{0}_{n+1}}{\delta K^{A}_{i}} - c^{A\alpha}_{i} \frac{\delta \tilde{S}^{0}_{n+1}}{\delta K^{\prime}_{i}} - c^{A\alpha +}_{i} \frac{\delta \tilde{S}^{0}_{n+1}}{\delta K^{\prime +}_{i}} - \frac{\delta \tilde{S}^{0}_{n+1}}{\delta u_{\alpha}} = 0,$$
$$- F^{B}_{\mu} \frac{\delta \tilde{S}^{0}_{n+1}}{\delta K^{B}_{\mu}} - c^{B}_{i} \frac{\delta \tilde{S}^{0}_{n+1}}{\delta K^{\prime}_{i}} - c^{B+}_{i} \frac{\delta \tilde{S}^{0}_{n+1}}{\delta K^{\prime +}_{i}} - \frac{\delta \tilde{S}^{0}_{n+1}}{\delta u} = 0. \qquad 39$$

It is obvious that Eq. (39) can be proved rigorously, and is consistent with Eq. (15). So, it can be used as a starting point for the higher *order renormalization*.

It should be noted that the first one of the equations (37) implies that $\tilde{S}^{0}_{n+1}$ is invariant under the following B.R.S. transformations (Eq.

10 is under the variant transformation):

$$\begin{aligned}
\delta A^{0}_{i(n+1)} &= (\Delta_i^{ab} + g^0_{(n+1)} t^b_{ij} A^0_{j(n+1)}) u^0_{b(n+1)} \delta\lambda^0, \\
\delta B^0_{\mu(n+1)} &= (\Delta^B_\mu u^0_{(n+1)}) \delta\lambda^0, \\
\delta s^0_{l(n+1)} &= \frac{-i}{2} (g^0_{(n+1)} \tau^b_{lk} s^0_{k(n+1)} u^0_{b(n+1)} + g'^0_{(n+1)} s^0_{l(n+1)} u^0_{(n+1)}) \delta\lambda^0, \\
\delta s^{+0}_{l(n+1)} &= \frac{i}{2} (g^0_{(n+1)} s^{+}_{k(n+1)} \tau^b_{kl} u^0_{b(n+1)} + g'^0_{(n+1)} s^{+0}_{l(n+1)} u^0_{(n+1)}) \delta\lambda^0, \\
\delta u^0_{a(n+1)} &= -\frac{1}{2} g^0_{(n+1)} f_{abc} u^0_{b(n+1)} u^0_{c(n+1)} \delta\lambda^0, \\
\delta \bar{u}^0_{a(n+1)} &= \xi^0_A (F^{Aa}_i A^0_{i(n+1)} + c^{Aa0}_{k(n+1)} s^0_{k(n+1)} + c^{Aa+0}_{k(n+1)} s^{+0}_{k(n+1)}) \delta\lambda^0, \\
\delta u^0_{(n+1)} &= 0, \\
\delta \bar{u}^0_{(n+1)} &= \xi^0_B (F^B_\mu B^0_{\mu(n+1)} + c^{B0}_{k(n+1)} s^0_{k(n+1)} + c^{B+0}_{k(n+1)} s^{+0}_{k(n+1)}) \delta\lambda^0.
\end{aligned} \tag{40}$$

Trying variant transformation again, $A^o_{i(n+1)}, B^o_{\mu(n+1)}, \ldots$ are transformed into $A_i, B_\mu, \ldots$ according to Eq. (34). Among those gauge fixed terms, $\xi_A^{1/2} F_i^{Aa} A_i$ and $\xi_B^{1/2} F_\mu^B B_\mu$ are not renormalized. For the B.R.S. transformation which is related to δu_a and δu, $\xi_A F_i^{Aa} A_i$ and $\xi_B F_\mu^B B_\mu$ are invariant before and after renormalization. Because of these, we should choose

$$\delta\lambda^0 = Z_3^{1/2} \tilde{Z}_3^{1/2} \delta\lambda = Z_3'^{1/2} \tilde{Z}_3'^{1/2} \delta\lambda.$$

Then we have:

$$\begin{aligned}
\delta A_i &= ((\tilde{Z}_3)_{n+1} \Delta_i^{ab} + g(\tilde{Z}_1)_{n+1} t^b_{ij} A_j) u_b \delta\lambda, \\
\delta B_\mu &= ((\tilde{Z}'_3)_{n+1} \Delta^B_\mu u) \delta\lambda, \\
\delta s_l &= \frac{-i}{2} (g(\tilde{Z}_1)_{n+1} \tau^b_{lk} s_k u_b + g'(\tilde{Z}'_1) s_l u) \delta\lambda, \\
\delta s_l^+ &= \frac{i}{2} (g s_k^+ (\tilde{Z}_1)_{n+1} \tau^b_{kl} u_b + g'(\tilde{Z}'_1) s_l^+ u) \delta\lambda, \\
\delta u_a &= -\frac{1}{2} g(\tilde{Z}_1)_{n+1} f_{abc} u_b u_c \delta\lambda, \\
\delta \bar{u}_a &= \xi_A (F_i^{Aa} A_i + c_k^{Aa} s_k + c_k^{Aa+} s_k^+) \delta\lambda, \\
\delta u &= 0, \\
\delta \bar{u} &= \xi_B (F^B_\mu B_\mu + c^B_k s_k + c_k^{B+} s_k^+) \delta\lambda,
\end{aligned} \tag{42}$$

where

$$s_l = \bar{s}_l + (v_l)_{n+1}, \qquad s_l^+ = \bar{s}_l^+ + (v_l^-)_{n+1}. \tag{43}$$

Here, (42) is the B.R.S. transformation related to Eq. (39) (s^c_{n+1} is invariant under the B.R.S. transformation), and is also the general

form of the B.R.S. transformation of (16). Replacing n+1 by n, (42) will become (16).

Comparison between (42) and (10):

(a) In Eq. (10), $s_l = \tilde{s}_l$, $s_l^* = \tilde{s}_l^*$, but in Eq. (42), s_l and s_l^* are translated by Eq. (43). This is necessary for the elimination of the divergent terms ν_l and ν_l^* (gauge related vacuum expectation value) which appeared during the renormalization process.

(b) Between Eqs. (10) and (42), the following relations hold:

$$
\begin{aligned}
\Delta_i^{Ab} &\to \Delta_{i(n+1)}^{Ab0} = (\tilde{Z}_3)_{n+1}\Delta_i^{Ab}, \\
t_{il}^{b} &\to t_{il(n+1)}^{b0} = (\tilde{Z}_2)_{n+1} t_{il}^{b}, \\
\tau_{lm}^{b} &\to \tau_{lm(n+1)}^{b0} = (\tilde{Z}_2)_{n+1}\tau_{lm}^{b}, \\
f_{abc} &\to f_{abc(n+1)}^{0} = (\tilde{Z}_1)_{n+1} f_{abc}.
\end{aligned}
\tag{44}
$$

In other words, Δ_i^{Ab} will become bare $\Delta_{i(n+1)}^{Ab0}$, ... etc. after the correction of (n+1) order perturbation. These bare Δ^0, t^0, τ^0, and f^0 still satisfy (from Eqs. (44) and (6))

$$
\begin{aligned}
& t_{il(n+1)}^{a0} t_{lk(n+1)}^{b0} - t_{il(n+1)}^{b0} t_{lk(n+1)}^{a0} = f_{abc(n+1)}^{0} t_{ik(n+1)}^{c0}, \\
& \left(-\frac{i}{2}\right)\tau_{kl(n+1)}^{a0}\left(-\frac{i}{2}\right)\tau_{lm(n+1)}^{b0} - \left(-\frac{i}{2}\right)\tau_{kl(n+1)}^{b0}\left(-\frac{i}{2}\right)\tau_{lm(n+1)}^{a0} \\
& \qquad = f_{abc(n+1)}^{0}\left(-\frac{i}{2}\right)\tau_{km(n+1)}^{c0}, \\
& t_{lm(n+1)}^{a0}\Delta_{m(n+1)}^{Ab0} - t_{lm(n+1)}^{b0}\Delta_{m(n+1)}^{Aa0} = f_{abc(n+1)}^{0}\Delta_{l(n+1)}^{Ac0}.
\end{aligned}
\tag{45}
$$

This is the same as Eq. (6).

(c) Exchanging i and l in Eq. (45), we have

$$
t_{il(n+1)}^{a0} t_{li(n+1)}^{b0} - t_{il(n+1)}^{b0} t_{li(n+1)}^{a0} = f_{abc(n+1)}^{0} t_{ii(n+1)}^{c0} = 0.
$$

$f_{abc} = M_{ab}^{c}$ can be treated as a matrix. For different c, M^c's are mutually independent. So, we have

$$
t_{ii(n+1)}^{c0} = 0. \tag{46}
$$

Since $f_{abc} \sim \varepsilon_{abc}$, so $f_{abc(n+1)}^{0} \sim \varepsilon_{abc}$. This implies

$$
f_{aac(n+1)}^{0} = 0. \tag{47}
$$

Therefore, under the B.R.S. transformations of Eq. (42), the sum of

the Jacobian diagonal of the functional integral volume element is zero and the Jacobian is 1. The Slavnov identical equations of $\tilde{\Gamma}$ at (n+1) order can thus be obtained. (i.e., Eq.(17), $\tilde{\Gamma}$ is derived by s^c_{n+1})

This will guarantee that the same renormalization process can be continued for the next higher orders.

(d) The gauge transformations which are corresponding to Eq. (42) are (let $u_b\delta\lambda = -\frac{1}{(\tilde{Z}_3)_{n+1}}\sigma^b,\ u\delta\lambda = -\frac{1}{(\tilde{Z})_{n+1}}\sigma$)

$$\delta A_i = (\Delta_i^{Ab} + \frac{(\tilde{Z}_1)_{n+1}}{(\tilde{Z}_3)_{n+1}} g\epsilon^b_{ij} A_j)\theta^b,$$

$$\delta B_\mu = (\Delta^B_\mu)\theta,$$

$$\delta s_i = \frac{-i}{2}\left(\frac{(\tilde{Z}_1)_{n+1}}{(\tilde{Z}_3)_{n+1}} g\tau^b_{ik}s_k\theta^b + \frac{(\tilde{Z}')_{n+1}}{(\tilde{Z}_3)_{n+1}} g' s_i\theta\right),$$

$$\delta s_i^+ = \frac{i}{2}\left(\frac{(\tilde{Z}_1)_{n+1}}{(\tilde{Z}_3)_{n+1}} g s_k^+\tau^b_{ki}\theta^b + \frac{(\tilde{Z}')_{n+1}}{(\tilde{Z}_3)_{n+1}} g' s_i^+\theta\right). \qquad 48$$

It can be seen from (b), (c), and (d) that the structure constants of gauge groups before and after renormalization are one to one correspondence, and that the Abel subgroups are still Abel subgroups. This is the proof of the isomorphism between gauge groups before and after renormalization.

In the presence of fermion fields ψ_L and ψ_R, since gA_i and $g'B_\mu$ appear in $\mathcal{L}_L$ and $\mathcal{L}_R$, they are zero after the operation by $\frac{A_i}{2}\frac{\delta}{\delta A_i} - \frac{g}{2}\frac{\partial}{\partial g}$, $\frac{B_\mu}{2}\frac{\delta}{\delta B_\mu} - \frac{g'}{2}\frac{\partial}{\partial g'}$. This will only give new renormalization constants Z_L and Z_R, but will not affect the overall renormalization. The fermion field coupling with the Higgs field will give new coupling constants and then new renormalization constants. But again this will not affect the overall renormalization.

If the vacuum spontaneous symmetry is broken, then the discussion of Zinn-Justin and Lee [2] is applicable. No discussion will be given here.

The method in this paper also can be applied to the case with several subgroups.

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ON THE ISOMORPHISM BETWEEN GAUGE GROUPS BEFORE AND AFTER RENORMALIZATION, IN THE PRESENCE OF ABEL SUBGROUPS AND HIGGS FIELDS

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ABSTRACT

We give a rigorous proof on the isomorphism between gauge groups before and after renormalization, in the presence of Abel subgroups and Higgs fields.

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